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Determination of the force of the actuators and constraint forces in the inverse dynamic analysis of mechanisms [†]

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Abstract

In the computer analysis of mechanisms it is usual to choose the actuators that drive them in terms of the ease with which they can be introduced into the definition of the system rather than in terms of describing the reality. Once the force of the replacement actuator used has been obtained, that of the real actuator must be determined and some of the calculated constraint forces differ from the real ones. This work studies the situation described in the inverse dynamic analysis of mechanisms of 1-DOF. It will show how to obtain the force of the real actuator from the force of the actuator used in the simulation process, and proposes the use of auxiliary mechanisms which aid in the description of the real actuators. It also provides criteria for determining which constraint forces obtained in the simulation process with the replacement actuator are different to the real ones.

Keywords: Actuators; Inverse dynamic analysis; Mechanism 1DOF; Simulation; Virtual work

1. Introduction

In the computer analysis and simulation of mechanisms it is normal to analyse different alternatives of actuators in order to produce the same movement in the mechanism [1-3]. In computer-based simulation the actuators are usually chosen in terms of the ease with which they can be introduced into the definition of the system rather than in terms of describing the reality [4]. Essentially, this replacement of actuators is done in the simulation of manually accionated mechanisms. In the simulation software, there is not always a menu with the adequate actuator or enough free parameters available for its definition to coincide with the reality. The possibility of using internal programming for the analysis applications, or linking up with external calculus applications, is not always available and is not easily done. If the study is not just kinematic the substitution of actuators involves two problems. On the one hand, the actuator action, moment or force, obtained from the simulation process differs from the action of the real actuator. On the other hand, some of the calculated constraint actions, moments or forces, differ from the real ones. It is extremely difficult to obtain the latter from the calculated ones.

This work will study, using the virtual work method presented in the virtual power version in [5], the situation described in the inverse dynamic analysis of mechanisms. It also provides criteria for determining which of the constraint forces obtained in the simulation process with the replacement actuator are different to the real ones. It will show how to obtain the action of the real actuator from the action of the actuator used in the simulation process, thereby avoiding the use of internal programming or links with other applications, if available.

Gallardo et al. [6] use the virtual work method to determine the generalized force of the actuators. The fact that the location of these generalized forces is unknown makes it impossible to determine the behaviour of the constraint forces. Geike and McPhee [7] carry out an inverse dynamic analysis of parallel manipulators and show that the constraint forces are not considered at all. Lu [8, 9] uses the virtual work method to determine, in spatial parallel manipulators, the generalized forces of the actuators and relates them to the real forces that they exert. It must be pointed out that different values of real forces may lead to the same generalized forces but this involves different values of the constraint forces.

Given the difficulty of calculating the real constraint forces from those obtained in the simulation, the use of auxiliary mechanisms is proposed, to enable the real description of the actuators with the elements available in the simulation programme and therefore, the direct calculation of all the constraint actions of the original mechanism.

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2. Analysis

2.1 Actuators

In the inverse dynamic study of a system of one degree of freedom, it is necessary to use one actuator which defines the temporal evolution of the mechanism. As the system has one degree of freedom, the distribution of velocities and accelerations is unique. Therefore, in a virtual movement compatible with the constraints, the virtual work associated with all the forces $\boldsymbol{F}_i$ and moments $\boldsymbol{M}_i$, Eq. (1), of interaction and inertia is independent of the actuator used to move the mechanism, as long as the same movement, velocity and acceleration are imposed, at any point of the mechanism. In this situation, the virtual work of any actuator is always the same, and as a result the action of one actuator can be calculated just by knowing that of another.

$$\delta W^* = \sum_i \boldsymbol{F}_i \cdot \delta \boldsymbol{r}_i^* + \sum_i \boldsymbol{M}_i \cdot \delta \boldsymbol{\varphi}_i^* = 0 \tag{1}$$

However, getting two different actuators to perform the same movement with the same velocity and acceleration in a mechanism is not easy, as in every instant one actuator must obey the geometric conditions imposed by the other, resulting in a system, in general, of non-linear equations corresponding to the geometrical constraint equations. Evidently, the simplest case is that of null movement because it does not need the calculation of the inertia forces nor, therefore, of the accelerations or the forces which depend on velocity. This situation applies to the quasistatic analysis in which the actuators are only used to keep the mechanism at rest in different configurations.

If, with one or another actuator, the same real movement can be obtained, applying the virtual work method with the only possible virtual movement compatible with the constraints allows to write for one or another actuator:

$$\delta W^*_{\text{act. 1}} + \delta W^*_{\text{rest of forces}} = 0 \tag{2}$$

$$\delta W^*_{\text{act. 2}} + \delta W^*_{\text{rest of forces}} = 0 \tag{3}$$

As the virtual work of the rest of the forces $\delta W^*_{\text{rest of forces}}$ is the same, given that the virtual and real movement are identical, the conclusion is that $\delta W^*_{\text{act. 1}} = \delta W^*_{\text{act. 2}}$. From this base, the force or torque of an actuator can be found when that of the other and the distribution of virtual displacements are known. Evidently, the distribution of virtual displacements coincides with the distribution of infinitesimal real displacements; therefore it can be determined from the kinematic analysis of the original mechanism by simple multiplying the velocities by d*t*.

The Watt six-bar mechanism has been studied by various authors [2, 10, 11]. In Fig. 1 the alternative to this mechanism is shown, as presented by Artobolevsky [12] with the name of Watt six-bar articulated mechanism of rectilinear guidance and which coincides with the definition given by various authors [13-15] for Stephenson III six-bar linkage. This mechanism can be driven by an angular actuator which controls the orientation of the OP bar or by a linear actuator in the direction of the TQ bar. The analysis of this mechanism via the angular actuator can be done without difficulty; however, the simulation, using the linear actuator applying the required force at point Q, is unavailable in the majority of programmes as a direct option in the actuators menu.

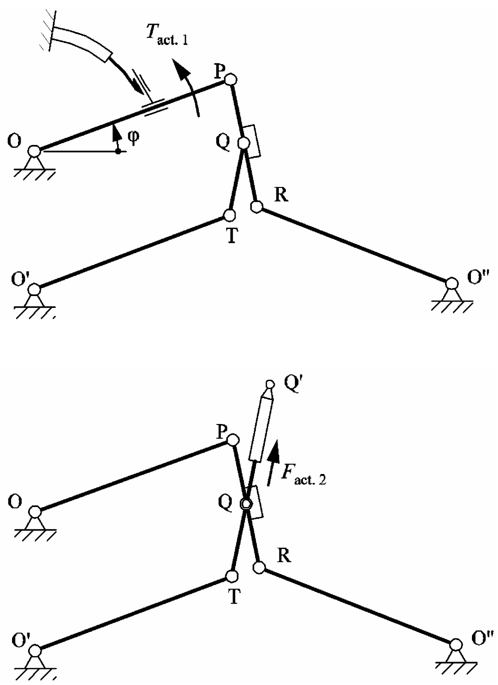

Fig. 1. Stephenson III six-bar linkage driven by an angular or a linear actuator.

The application, as has been shown, of the virtual work method, in its derivative version (virtual power method), on the six-bar mechanism in Fig. 1 leads to:

$$T_{\text{act.1}}\,\omega_{\text{OP}} = \boldsymbol{F}_{\text{act2}} \cdot \boldsymbol{v}_{\text{rel}}(\text{Q})$$

$$T_{\text{act.1}} = \boldsymbol{F}_{\text{act.2}} \cdot \frac{\boldsymbol{v}_{\text{rel}}(\text{Q})}{\omega_{\text{OP}}}$$

$$T_{\text{act.1}} = F_{\text{act.2}} \cdot \frac{\text{d}\mathbf{Q'Q}}{\text{d}\varphi} \tag{4}$$

where φ is the angle of orientation of the OP bar with respect to the horizontal. The expression (4) shows clearly that the relationship between the actions of the two actuators used is due exclusively to the configuration.

2.2 Constraint actions

When the virtual work method is used to find a component of constraint force or moment, the kinematic restriction corresponding to the said component may be substituted, conceptually, by an actuator which guarantees the same kinematic condition –a constraint actuator– (Fig. 2). By substituting a constraint for a constraint actuator the mechanism gains one degree of freedom. The constraint force or moment can be found using the virtual work method by making a virtual movement compatible with the rest of the constraints and defined by the virtual evolution of the constraint actuator (virtual motion non-compatible with the substituted constraint). The constraint action may differ according to the driving actuator used in the mechanism.

In order to compare the constraint actions obtained with two distinct driving actuators imposing the same real movement, when applying the virtual work method the driving actuator is virtually fixed and a virtual movement defined by the virtual evolution of the constraint actuator is performed. Carrying out this type of virtual movement is appropriate as with it the virtual work of the driving actuator is null, which aids the interpretation of the result. If the virtual work of the rest of forces and moments, including those of inertia, is the same with each of the aforementioned driving actuators, the constraint force or moment does not depend on the driving actuator. Otherwise, the constraint action will depend on the driving actuator.

If, for example, two driving actuators are used, applied alternatively to the same solid jointed to the frame and it is necessary to find the action of any other constraint of the mechanism which is different to the said joint, the proposed virtual movement non-compatible with the constraint is unique and therefore the constraint force is also unique. The virtual movement non-compatible with the constraint is unique because by keeping one or the other of the two actuators still the solid jointed to the frame also remains still.

The content of the previous paragraph is made clear in the study of the vertical constraint force F_v in the fixed joint R of

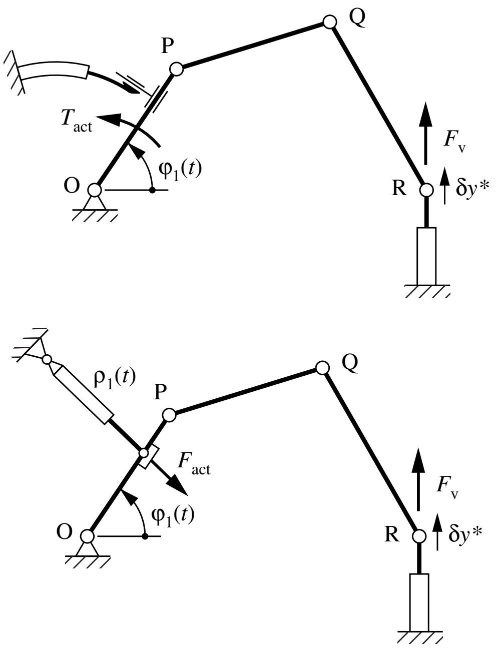

Fig. 2. Four-bar linkage driven by an angular actuator and a linear actuator.

the four-bar linkage in Fig. 2. The movement of the four-bar linkage is studied by actuating on the orientation of the OP bar, either by an angular actuator that varies the angle φ_1 (t) or a lineal actuator that varies the distance ρ_1 (t). Just as the application of the virtual work method shows, the value of F_v does not depend on the actuator used. The null condition of vertical velocity of R is imposed by a constraint actuator.

With the angular actuator and the virtual movement $\delta\varphi_1^*=0$ and $\delta y^* \neq 0$:

$$\underbrace{T_{act} \cdot \delta\varphi_1^*}_{=0} + F_v \cdot \delta y^* + \delta W^*_{\text{rest of forces}} = 0 \tag{5}$$

With the linear actuator and the virtual movement $\delta\rho_1^*=0$ and $\delta y^* \neq 0$:

$$\underbrace{F_{act} \cdot \delta\rho_1^*}_{=0} + F_v \cdot \delta y^* + \delta W^*_{\text{rest of forces}} = 0 \tag{6}$$

The virtual work of the rest of the forces $\delta W^*_{\text{rest of forces}}$ is the same in both cases, as by annulling the virtual movement of the OP bar the real and virtual kinematics of the rest of the mechanism are identical in both cases. As a result, F_v does not depend on the actuator.

The coincidence of the constraint actions, with actuators that produce the same real movement of the system, is verified for all the constraints which do not kinematically condition the movement of the solids acted by the driving actuators –non-conditioning constraints. These solids and the other elements – solids and constraints– which kinematically condition the movement of the acted solids define a sub-chain of one degree of freedom. This is justified because if there was more than one degree of freedom the rest of the elements in the mechanism should condition the movement of the sub-chain, given that the studied mechanism has only one degree of freedom. If the virtual work method is applied without one of the non-conditioning constraints the system has two degrees of freedom in its virtual movement. With the first virtual degree of freedom the driving actuator is stopped and with it all of the sub-chain, irrespective of the actuator. With the second virtual degree of freedom the suitable movement is imposed in order to obtain the constraint action. This virtual movement is unique and therefore so is the constraint action, as is clear from the expressions (5) and (6) in the previous example.

In short, the substitution of actuators does not affect the constraints, which may be removed without affecting the kinematics of the links of the mechanism which are directly acted by the actuators. These constraints coincide with those of the Assur groups [16, 17] added to the minimum kinematic chain which defines the movement of the acted links.

If, for example, in the four-bar linkage shown in Fig. 3 the movement of the PQ bar is controlled, instead of that of the OP bar, its movement can be guided either through an angular actuator which varies the angle φ_2 (t) or through a linear actuator which varies the distance ρ_2 (t). In this case, as is shown

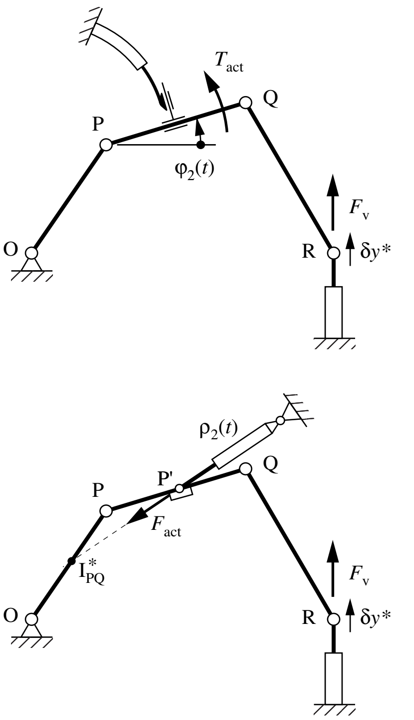

Fig. 3. Four-bar linkage driven by an angular actuator and a linear actuator.

from the application of the virtual work method, the value of F_v does depend on the actuator used.

With the angular actuator and the virtual movement $\delta\varphi^*_2 = 0$ and $\delta y^* \neq 0$ the PQ bar performs a virtual movement of translation and $\delta \boldsymbol{r}^*(\mathrm{P}) = \delta \boldsymbol{r}^*(\mathrm{Q})$.

With the linear actuator and the virtual movement $\delta\rho^*_2 = 0$ and $\delta y^* \neq 0$ the PQ bar performs a virtual movement of rotation around its virtual instantaneous centre of rotation, point $\mathrm{I}^*_{\mathrm{PQ}}$, which can be found through the application of the theorem of the three centrers or of Aronhold-Kennedy [13, 14]. In this case $\delta \boldsymbol{r}^*(\mathrm{P}) \neq \delta \boldsymbol{r}^*(\mathrm{Q})$.

The virtual movements of the three bars are different in the two virtual movements and, although $T_{act} \cdot \delta\varphi^*_2 = 0$ and $F_{act} \cdot \delta\rho^*_2 = 0$, the virtual work associated with the rest of the forces $\delta W^*_{\text{rest of forces}}$ is, in principle, different; therefore F_v does depend on the actuator used.

Analysis of the two cases studied in the four-bar linkage of Fig. 2 and Fig. 3 shows how the constraint force F_v in the revolute joint R does not depend on the driving actuator when this acts on the OP bar, as this constraint does not influence the movement of the said bar, but it does depend on the driving actuator when this acts on the PQ bar, as the constraint at R does influence the movement of PQ.

Having analysed the difficulty of relating some of the constraint actions caused by different actuators it is suggested that, if they are to be calculated, the real actuator should be conveniently modelled, although doing this involves a certain dose of ingenuity and is a laborious task. In this work two examples are shown, one of which is the case of using alongside the original mechanism another mechanism of identical kinematics superimposed on it (the alias mechanism).

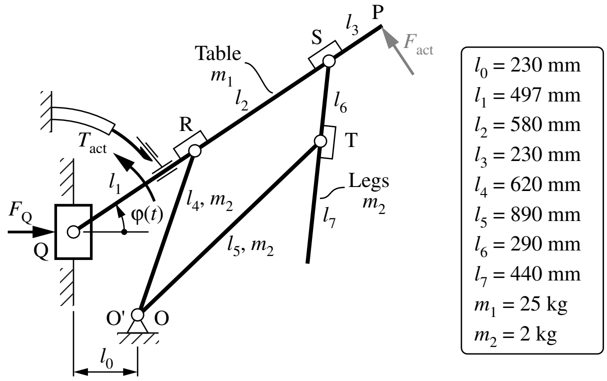

Fig. 4. Schematic of the mechanism of a wing of the ping-pong table.

3. Simulation and results

In carrying out the proposed simulations, the PAM – Programme of Analysis of Mechanisms (only available in Spanish and Catalan)– application was used, which performes the static, kinematic and kinetostatic analysis of planar mechanisms of one or more degrees of freedom driven by the same number of angular or linear actuators [18, 19]. This application obtains and allows one to export, for a set of time instants, all the kinematic and dynamic variables of the mechanism, which facilitates the obtention and presentation of the results of the planned examples.

As an application of what was stated in previous sections, the quasi-static closure of a wing of a ping-pong table was studied as shown in Fig. 4, which is a material form of the Stephenson III six-bar mechanism. All the bars are assumed to be uniform with the centre of inertia at the mid-point between their respective extremes. The simulation is carried out on this mechanism to obtain the human driving force F_{act} applied perpendicularly to the bar at point P, but the simulation programme lacks a suitable actuator. Therefore the mechanism was modelled using an angular actuator which controlled the angle φ that the PQ bar forms in respect to the horizontal. One of the results obtained in the simulation process is the torque T_{act} produced by the angular actuator.

The application of the virtual work method with a virtual movement $\delta\varphi^*$ compatible with the constraints and making a distinction between the virtual work of the actuator and the work of the rest of the forces results in:

$$\delta W^*_{act} + \delta W^*_{\text{rest of forces}} = 0 \tag{7}$$

As the aim was to obtain the same kinematics with the angular actuator as with the linear actuator that produced the human driving force F_{act}, as shown in section 2, the work of the rest of the forces was the same in both cases; therefore the virtual work of the torque of the angular actuator and that of

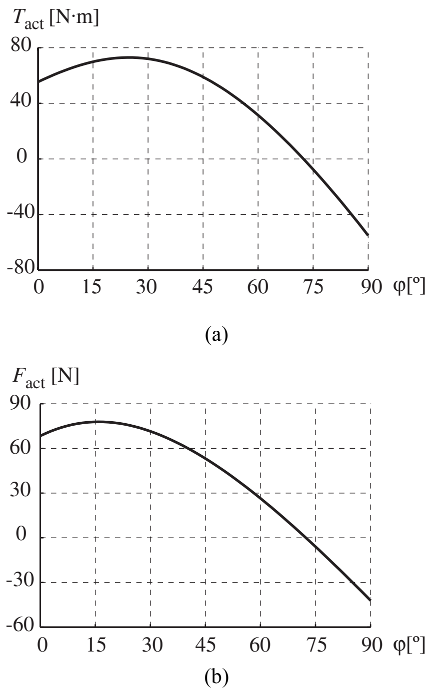

Fig. 5. Ping-pong table: torque applied by the angular actuator (a) and force applied at point P (b).

the force of the linear actuator should also be the same. And so:

$$T_{\mathrm{act}}\ \delta\varphi^* = \boldsymbol{F}_{\mathrm{act}} \cdot \delta\boldsymbol{r}^*(\mathrm{P}) \ \rightarrow\ F_{\mathrm{act}} = \frac{T_{\mathrm{act}}\ \delta\varphi^*}{\delta r^*(\mathrm{P})\big|_{\perp\mathrm{QP}}} = \frac{T_{\mathrm{act}}\ \omega_{\mathrm{QP}}}{v(\mathrm{P})\big|_{\perp\mathrm{QP}}} \tag{8}$$

Based on the ω_{QP}, $\boldsymbol{v}(\mathrm{P})$ and T_{act} obtained through the simulation, we can determine (8) the necessary human driving force, F_{act}, which should be applied at point P to close the ping-pong table. The kinematics of the table wing, QP, is unequivocally defined by the guide Q and the revolute joints R and O', while the revolute joints S, T and O may be eliminated without affecting the movement of the wing. Therefore, the actions in the revolute joints S, T and O do not depend on the actuator used, whereas the constraint actions of the revolute joints R, O' and the guide Q do.

Fig. 5(a) shows the torque needed to close the wing of the ping-pong table when this is done with the angular actuator. Fig. 5(b) shows the necessary perpendicular force applied at point P to close it, obtained by applying the virtual work method. Fig. 6 shows the normal constraint force on the guide F_{Q} (the continuous line shows values obtained with the angular actuator, the dashed line shows values obtained by applying perpendicular force on the table at point P).

After analysing the situation, in order to deal with many cases of actuator substitution in which all the constraint actions and those of the real actuator had to be correctly calculated, the use of auxiliary mechanisms was proposed, which would help in manipulating the mechanism by reproducing the real actuator. One particular, and common, situation would be the use of an alias mechanism of identical kinematics to the original. These mechanisms may be of negligible inertia or not, according to need, are moved by an actuator and interact with the original mechanism through an additional constraint or actuator which goes on to generate the force or moment of the real actuator as originally planned.

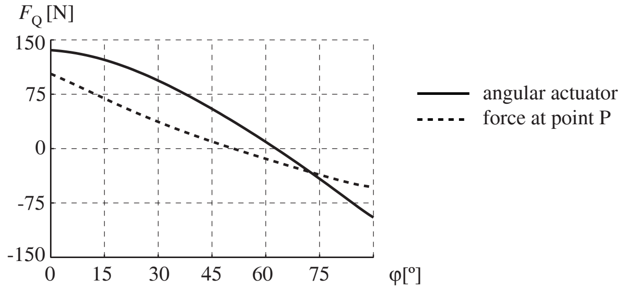

Fig. 6. Ping-pong table: normal force at the sliding-joint constraint.

In short, the original actuator is modelled through the auxiliary mechanism and the constraint (or actuator) of this with the studied mechanism. So, it is not a case of substituting one actuator for another; it is a question of conveniently modelling the original actuator using, often ingeniously, a series of tricks without needing to rely on internal programming modules or links to external calculation programmes. In this way, the constraint forces and moments of the original mechanism as well as those of the actuator (constraint force or moment between the two mechanisms, original and auxiliary, or the force of the actuator interposed between the two) can be obtained directly.

As a first example, there is the study and simulation of the quasistatic opening of a shelf, as shown in Fig. 7. The only element considered to have mass is the uniform QP bar, with its centre of inertia at the mid-point between Q and P. The corresponding alias mechanism is a duplicate of the original and between the two there is a pin-joint constraint. This ensures that they both move identically and avoids creating redundancies. The guide is located in the alias mechanism and the pin is in the original mechanism. The guide has a constant orientation respecting the alias mechanism and therefore also in terms of the original mechanism. In short, the constraint force F_{P} in the guide is of a constant direction and represents the constant driving force, perpendicular to the shelf, that a user should make to open it out (Fig. 8). In the simulation process the alias mechanism, of negligible inertia or not, is driven by an angular actuator which controls the angle φ of the OQ alias bar of the aforementioned mechanism.

The discontinuity in the necessary force F_{P} shown in Fig. 8 corresponds to the configuration in which the point P only has velocity in the direction of the QP bar. This configuration corresponds to that in which the extensions of the O'R and OQ bars are cut at the same point with the line perpendicular to QP which passes through P. This intersection point is the

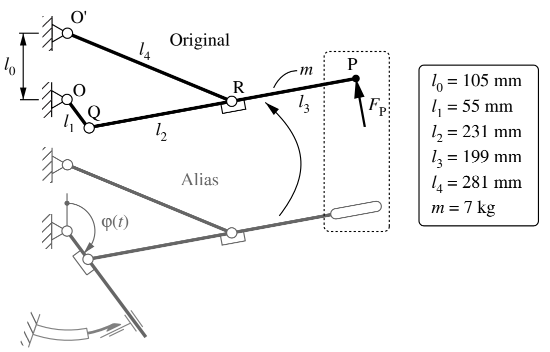

Fig. 7. Schematic of a folding shelf. Original and alias mechanisms.

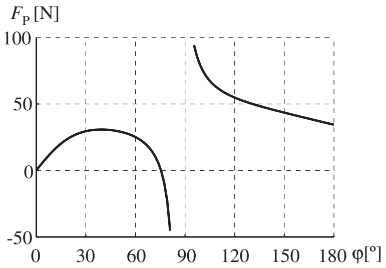

Fig. 8. Necessary perpendicular force at point P to open the shelf.

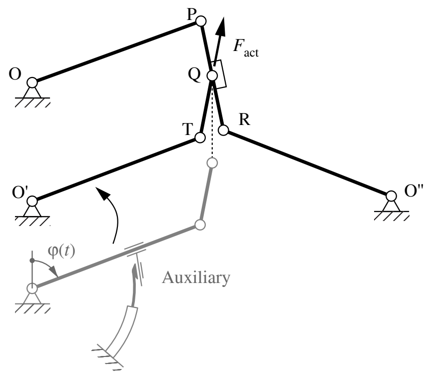

Fig. 9. Stephenson III and auxiliary mechanisms with actuator in the direction TQ.

instantaneous centre of rotation of the QP bar, as shown by applying the theorem of the three centres to the O'R, OQ and QP bars. In this configuration the velocity of P is perpendicular to F_P and so the coordinate driving the actuator, that represents the force exerted by a user, reaches a dead point.

As a second example, in Fig. 9 the Stephenson III mechanism from Fig. 1 is shown with the auxiliary mechanism that reproduces the force of the linear actuator in the direction of the TQ bar. All the bars of the original mechanism are assumed to be uniform with the centre of inertia at the mid-point. For the force of the simulated actuator to be in the direction of the bar, all the bars of the auxiliary mechanism should be of negligible inertia. Fig. 10 shows the force of the linear actuator in the direction of the TQ bar.

The auxiliary mechanisms allow to obtain the action of some actuators that are unavailable in the majority of the

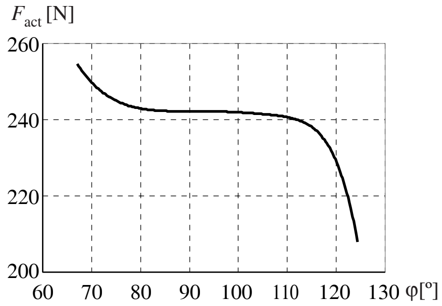

Fig. 10. Force of the linear actuator in the direction of the TQ bar.

simulation programmes. The obtained results are the same as using the replacement of actuators with the advantage that it is not necessary to do calculations after the simulation.

4. Conclusions

The use of the virtual work method proved useful in the post-processing of the inverse dynamic analysis of mechanisms of one degree of freedom, particularly in the substitution of actuators, as the example of the wing of the ping-pong table shows.

If the real movement (velocities and accelerations) of a mechanism of one degree of freedom is the same through the use of two different actuators, a constraint action does not depend on the actuator when, eliminating the constraint, the kinematics of the elements driven directly by the actuator are not affected.

The use of auxiliary mechanisms or alias mechanisms in analysing mechanisms with computer programmes: increases the possibilities for choosing actuators, permits the satisfactory simulation of non-trivial actions on the mechanisms and simplifies considerably the determination of the constraint actions, as the examples of folding shelf and the Stephenson III mechanism show.

In conclusion, the two proposed methods (the replacement of actuators and the use of auxiliary mechanisms) can achieve both the action of the real actuator and the constrain actions.

The replacement of actuators needs to post-process the results of the simulation and it is not trivial to obtain the actual values of the constrain actions. In contrast, the use of auxiliary mechanisms provides the actual results, but the difficulty lies in finding out the right auxiliary mechanism.

Nomenclature

$\boldsymbol{F}_{\mathrm{act}\,i}$	:	Force of the actuator i
$\boldsymbol{F}_i$	:	Force
I_i^*	:	Virtual centre of rotation of the link i
$\boldsymbol{M}_i$	:	Moment
$T_{\mathrm{act}\,i}$	:	Torque of the actuator i
$\boldsymbol{v}$ (P)	:	Velocity of the point P
$v(\mathrm{P})\big\|_{\perp\mathrm{QP}}$	:	Velocity of the point P perpendicular to QP direction

$\boldsymbol{v}_{rel}(Q)$: Relative velocity of the point Q
$\delta \boldsymbol{r}_i^*$: Virtual linear displacement
$\delta r^*(P)|_{\perp QP}$: Virtual displacement of the point P perpendicular to QP direction
δW^* : Virtual work
δy^* : Virtual linear displacement in vertical direction
$\delta \boldsymbol{\varphi}_i^*$: Virtual angular displacement
$\delta \rho_i^*$: Virtual linear displacement in actuator direction
φ_i : Angular coordinate of the link *i*
ρ_i : Linear coordinate of the actuator *i*
ω_i : Angular velocity of the link *i*

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